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## V. An Experiment in Search of a Directive Action of one Quartz Crystal on another.

### By J. H. POYNTING, Sc.D., F.R.S., and P. L. GRAY, B.Sc.

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SINCE so many of the physical properties of crystals differ along the different axes, our ignorance of the nature and origin of gravitation allows us to imagine that the gravitative field of crystals may also differ along those axes. Dr. A. S. MACKENZIE ('Phys. Rev.,' vol. 2, 1895, p. 321) has described an experiment in which he failed to find any such difference. Using Boys's form of the Cavendish apparatus, he showed that the attraction of calc-spar crystals on lead and on other calc-spar crystals was independent of the orientation of the crystalline axes within the limits of experimental error-about one-half per cent. of the total attraction. He further showed that the inverse-square law holds in the neighbourhood of a crystal, the attractions at distances 3.714 centims., 5.565 centims., and 7.421 centims. agreeing with law to one-fifth per cent.

One of the authors of this paper had already pointed out ('The Mean Density of the Earth,' 1894, p. 7) that if the attraction between two crystal spheres were different for a given distance, according as their like axes were parallel or crossed, such difference should show itself by a directive action on one sphere in the field of the other. This directive action is suggested by the growth of a crystal from solution, where the successive parts are laid down in parallel arrangement----a fact which which we might perhaps interpret on the molecular hypothesis as showing that, within molecular range at least, there is directive action.

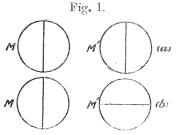
The experiment now to be described is a modification of one indicated in the work above referred to, carried out for two quartz spheres, and we may say at once that we have certainly not succeeded in proving the existence of a directive action of the kind sought for.

To bring out the principle of the method, let us suppose that the law of the attraction between two spheres with their like axes parallel, as in fig. 1 (a), is  $\text{GMM}'/r^2$ , where M, M' are the masses, r the distance between the centres, and G a constant for this arrangement. Let us further suppose that the law of attraction when the axes are crossed, as in fig. 1b, is  $G'MM'/r^2$ , where G' is a constant for this arrangement, and different from G.

Let us start with the spheres r apart, as in fig. 1 (a). The work done in removing 17.1.99

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M' to an infinite distance, in a line perpendicular to the parallel axes, is GMM'/r. Now turn M' through 90° to cross the axes, and bring it back to the original position, but with the axes crossed.



The force will do work G'MM'/r Then turn M' through 90° into its original orientation. Assuming that the forces are conservative, the total work vanishes, so that there must be a couple acting during the last rotation, which does work equal to the difference between the works done on withdrawal and approach.

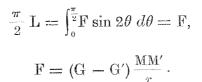
If we take the average value of the couple as L, then

$$\frac{\pi}{2}$$
 L = (G - G') $\frac{MM'}{r}$ .

Our suppositions as to the law of force are doubtless arbitrary, but they serve to show the probability of the existence of a directive couple accompanying any axial difference in the gravitative field.

In the absence of any distinction between the ends of an axis we may assume that the couple is "quadrantal," that is, that it goes through its range of values with the rotation of the sphere through  $180^{\circ}$  and vanishing in every quadrant, and we shall suppose that it is zero when the crystals are in the positions shown in fig. 1 (*a*), and fig. 1 (*b*).

Taking the couple as a sine function of amplitude F, we have



whence

But it is conceivable that the two ends of an axis are different, having polarity of the magnetic type. The couple would then be "semicircular," going through its range of values once and vanishing twice in the revolution. We shall suppose that the couple is zero when the axes are parallel. We should now have G and G' constants for the axes parallel, the one when like ends are in the same direction, the other when they are in opposite directions, and we have

$$\pi \mathbf{L} = (\mathbf{G} - \mathbf{G}') \frac{\mathbf{M}\mathbf{M}'}{r}$$

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But if F is the amplitude of the couple

and

$$\pi \mathbf{L} = \int_0^{\cdot} \mathbf{F} \sin \theta \, d\theta = 2\mathbf{F},$$
$$2\mathbf{F} = (\mathbf{G} - \mathbf{G}') \frac{\mathbf{M}\mathbf{M}'}{r}$$

To seek for the directive action we have made use of the principle of forced oscillations, thereby obtaining to some extent a cumulative effect, and at the same time largely eliminating the errors due to accidental disturbances.

Briefly the method was as follows :—A small quartz sphere, about 0.9 centim in diameter, was carried in a frame to which a light mirror was attached, and suspended by a quartz fibre inside a brass case, the position being determined by the reflection of a scale in the usual way. The complete time of torsional vibration was about 120 seconds.

Outside the case was a larger quartz sphere, about 6.6 centims. in diameter, its centre being level with that of the suspended sphere, and 5.9 centims. from it. The larger sphere could be rotated about a vertical axis through its centre at any desired rate. The crystalline axes of both were horizontal, that of the smaller sphere being perpendicular to the line joining the centres.

To test for the quadrantal couple, the larger sphere was rotated once in 230 seconds—a period nearly double that of the smaller sphere. To test for the semicircular couple, the larger sphere was rotated once in 115 seconds, or nearly the period of the smaller sphere.

Assuming that a couple exists, a continuous rotation of the larger sphere would set up a forced oscillation in the smaller sphere of the same period as the couple, and since the damping was very considerable, this forced oscillation would soon rise to approximately its full value. Meanwhile, any natural vibrations of the suspended system would be rapidly damped out. Though continually renewed by disturbances due to convection-currents and tremors, they would be irregularly distributed, and there was no reason to suspect that their maximum amplitude would recur at any particular phase of the period of the applied couple. To secure the distribution of successive maxima of natural vibrations of the smaller sphere over all phases of the forced period, the latter was made sensibly different from the natural period in the ratio 23:24; and though the cumulative effect of the forced oscillations was reduced by the largeness of this difference, we did not think it advisable to make the periods more nearly coincident, lest the distribution of the disturbances, which were sometimes large, should not be sufficient. This conclusion was arrived at from the results of preliminary experiments with more nearly equal periods.

During each complete period of the supposed applied couple, the position of the smaller sphere was read ten times at equi-distant intervals of time, and the scalereadings were entered in ten parallel columns, one horizontal line for each period. The

observations were continued usually for 70 or 80 periods. Adding up the columns and dividing by the number of periods, any forced oscillation would be indicated by a periodicity in the quotients. The periodicities found were too irregular to be taken as evidence of the existence of a couple.

## Description of the Apparatus.

The quartz spheres were placed in a cellar at Mason College, Birmingham, below the room in which the observing telescope and rotating apparatus were fixed.

The smaller sphere, 0.9 centim. diameter and weighing 1.004 grams, was held in an aluminium wire cage, and was suspended by a long, fine quartz fibre in a brass case from a torsion-head at the top of the case.

A light plane mirror was fixed to the cage, and opposite this mirror was a glass window in the case; in front of the window was a plane mirror at 45°, by means of which the light from the scale was reflected into the case and back again to the telescope, as shown in fig. 2.

The case was surrounded by a double-sided wooden box, lined within and without with tin-foil, and with cotton-wool between its inner and outer walls. The box was supported on indiarubber blocks to lessen tremors.

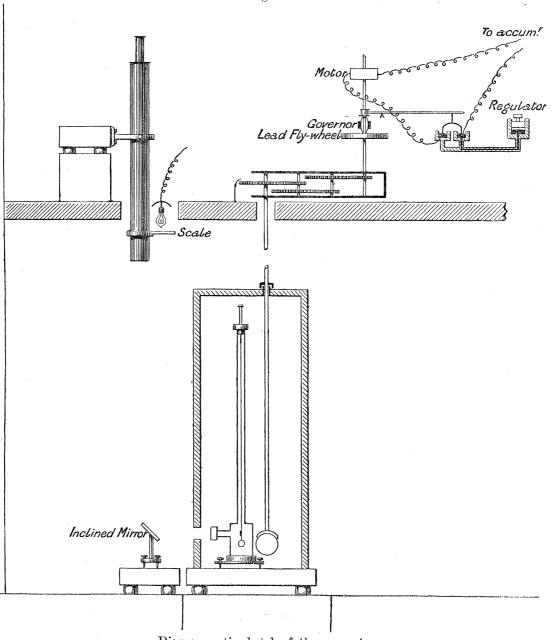
The larger sphere, 6.6 centims. diameter and weighing 399.9 grams, was held at the lower end of a vertical brass tube which terminated in a very carefully turned shallow brass bell, in which the sphere was held by tapes. The tube passed upwards through the top of the wooden casing without contact, a kind of air stuffing-box indicated in the figure serving to prevent currents through the hole. The tube came into the room above, and was there connected with a train of wheels, driven by an electromotor, the rotation of the motor being geared down from 1000 to 1. The observing telescope was fixed to a heavy stone slab resting on indiarubber blocks. standing on a brick-pillar, which was built on the brick arches forming the cellarroof. A diagonal scale (of half-millimetre graduations, divided into tenths by the diagonal ruling) was clamped to the telescope-tube and illuminated by an incandescent lamp, aided by a concave mirror. A tenth of a division could be read with certainty, and as the distance from scale to mirror was 358 centims., the position of the suspended sphere could be determined within a little more than one second of arc.

The steady rotation of the larger sphere was maintained by a regulator, for which we are indebted to Mr. R. H. HOUSMAN. It consisted of two parts:—(1) the governor proper, which automatically maintained approximate steadiness, and (2) a fine hand-adjustment, by which the motion could be accelerated or retarded when it got "out of time."

One lead to the motor went through two mercury-cups, and the circuit was completed by a fork of platinum-wire dipping into the cups. This wire was fastened

to one end of a wooden lever, the other end of which was attached to a sliding collar on the axle of the motor. To this collar were fastened the upper ends of the loaded springs of the governor, as shown in the figure. If the speed increased, the loads

Fig. 2



Diagrammatic sketch of the apparatus.

flying out pulled the collar down and so raised the wire out of the mercury-cups, and broke the circuit. As the speed diminished, the wire again dipped into the mercury and re-established the current. To diminish sparking the mercury was covered with alcohol, and the two cups were permanently connected by a high resistance shunt.

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The fine hand-adjustment consisted of a small wooden plunger working in a tube connected with one of the mercury-cups; by means of a screw the plunger could be raised or lowered, and the level of the mercury in the cup varied accordingly.

If the revolving sphere was found to be gaining or losing, it was quite easy to bring it "up to time" again by working the screw of the plunger.

The last of the train of driving-wheels was fixed on the tube supporting the larger sphere; its rim was divided into equal parts by numbered marks, the use of which will be explained directly. There were 20 numbered marks, at 18° interval; of these only 10 alternate ones were used for the quicker rotation, while the whole 20 were used for the slower speed.

## The Observations.

Two observers were required, one at the telescope to note the position of the smaller sphere, the other to regulate the speed of rotation of the larger sphere, and to notify when readings were to be taken by the first observer. The motion having been started, and brought to about the right speed, a time-table was rapidly prepared, showing the times, on the chronometer used, at which each of the numbered marks above mentioned should pass a fixed mark throughout the whole set of observations for one occasion. A signal was given at each passage of a mark past the fixed point, the observer at the telescope putting down the simultaneous scale-reading in a manner which will be understood from Table I., which may serve as a typical record. It does not appear to be necessary to give the full details in other cases. If the motion did not keep to the time-table, it was easily corrected by the hand adjustment already described.

Every reading in the same column is taken at the same phase in the rotation of the larger sphere, and therefore the mean readings of the columns should preserve any periodicity in the motion of the smaller sphere equal to that of the larger sphere, and more or less eliminate all others. These mean readings are given at the foot of Table I., and appear to indicate a slight periodic vibration, but this might be due to a want of symmetry in the larger sphere and its attachments about its axis of rotation, since the system supporting the smaller sphere and mirror was necessarily not symmetrical. The observations for each couple were on this account divided into two sets : for the semicircular couple the larger sphere was in the second set turned through 180° about a vertical axis from its position in the first set; for the quadrantal couple the rotation was 90°. For the final results the means of the results of the two sets were taken, in each case after the second set had been advanced by an amount corresponding to the change of position of the sphere.

Table II. contains all the mean results obtained in the same way as the figures at the foot of Table I., the greatest range being given in the last column as an indication of the magnitude of the disturbances.

In Table III. are given the means for each azimuth of the larger sphere in its support, the B and D series being advanced as mentioned above.

In combining the results it appeared useless to attempt to weight them according to the number of periods taken, since no accurate conclusion could be expected. 1twill be seen that in each case there is an outstanding periodicity, but the amplitude is less when the disturbances (as indicated by the greatest range during a period) are less, and it diminishes when the results are combined so as to lessen the effect of want of symmetry.

In the "quadrantal" observations (Series C, D), where the effect of want of symmetry of the apparatus should almost be eliminated, since it is approximately semicircular, the mean range is much smaller than in Series A and B.

For these reasons we do not think that our observations can be taken as indicating the existence of a couple of the kind sought, but only as giving a superior limit to its value, should it exist.

We now proceed to the Calculation of Superior Limit of Couple.

## Equation of Motion of the Smaller Sphere.

Let I be the moment of inertia of sphere and cage.

torsion couple per radian.  $\mu$ • • ,,

damping couple per unit angular velocity. λ ,,

, F cos pt be the supposed couple due to the larger sphere, having period  $2\pi/p$ . Then

$$I\theta + \lambda\theta + \mu\theta = F \cos pt.$$

Putting

$$\kappa = \lambda/I$$
;  $n^2 = \mu/I$ ;  $\mathrm{E} = \mathrm{F}/\mathrm{I}$ 

we have

The solution of this is

$$\theta = \frac{\mathrm{E}\sin\epsilon}{p\kappa} \cos\left(pt - \epsilon\right) + \mathrm{A}e^{-\frac{1}{4}\kappa} \cos\left\{\sqrt{\left(n^2 - \frac{1}{4}\kappa^2\right)t - \alpha}\right\}}.$$
 (2)

where  $\tan \epsilon = \frac{p\kappa}{n^2 - p^2}$  and A,  $\alpha$  are constants.

The first term in the value of  $\theta$  in (2) gives the forced, and the second term the natural vibrations, the period of the latter being

$$\frac{2\pi}{\sqrt{(n^2-\frac{1}{4}\kappa^2)}}=\text{T, say.}$$

The value of T was always very near to 120 secs., and the mean of various determinations during the observations gave

$$T = \frac{2\pi}{\sqrt{(n^2 - \frac{1}{4}\kappa^2)}} = 120.8 \text{ secs.} \qquad (3).$$

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Value of  $\kappa$ .—When there are only natural vibrations

 $\frac{\text{any complete swing}}{\text{next complete swing}} = e^{\frac{1}{3}\kappa \cdot \frac{1}{2}\mathrm{T}}.$ 

The value of this ratio was usually near 1.4. The mean of a number of determinations taken at various times was 1.3953. Putting

> $e^{30\cdot 2\kappa} = 1\cdot 3953,$  $\kappa = 0\cdot 011033.$

Value of n.—Substituting for  $\kappa$  in the value of T in (3) we get

and

we get

n = 0.052306.

 $n^2 = 0.0027359$ 

Value of  $\epsilon$ .—The forced period  $2\pi/p$  was always 115 secs., whence

$$\tan \epsilon = \frac{p\kappa}{n^2 - p^2} = 2.420,$$
$$\epsilon = 67^{\circ} 33',$$
$$\sin \epsilon = 0.9242.$$

From equation (1) it will be seen that the steady deflection due to F is  $\frac{E^2}{n}$  while from (2) the amplitude of the forced oscillations is  $\frac{E \sin \epsilon}{p\kappa}$  or  $\frac{n^2 \sin \epsilon}{p\kappa} \cdot \frac{E}{n^2}$ .

Using the values found for  $n\kappa$  and  $\epsilon$  we have

$$\frac{n^2\sin\epsilon}{p\kappa} = 4.196$$

or the forced oscillations give a cumulative effect, about four times the steady deflection due to the couple at its maximum value.

Value of Moment of Inertia, I.—This was found by vibrating the cage hung by a short quartz fibre, (1) when empty, (2) when containing the sphere, the times of vibration being respectively 8.38 secs. and 11.22 secs. The sphere weighs 1.004 grams, and its radius is 0.45 centim., so that its moment of inertia  $\frac{2}{5}$  Mr<sup>2</sup> = .08132.

From this, and the times of vibration, we get

$$I = 0.1821.$$

Value of F.—The vibrations were observed in scale divisions, each 0.05 centim., the distance between mirror and scale being 358 centims. If N is the number of scale divisions in the amplitude of vibration, *i.e.*, in half the range, we have from (2)

whence

$$\frac{\mathrm{E}\,\sin\,\epsilon}{p\kappa} = \frac{5\mathrm{N}}{2\times35800},$$
$$\mathbf{F} = \mathrm{EI} = 0.8293\mathrm{N}\times10^{-8},$$

using the values already found for  $\epsilon$ ,  $\kappa$ , I.

Taking the limiting values of the amplitudes as half the mean ranges given in Table III., the vibration due to the quadrantal couple has amplitude not greater than 0.033 div., and that due to the semicircular couple, amplitude not greater than 0.095 div. Whence

and

F (quadrantal) is not greater than  $2.737 \times 10^{-10}$ ,

F (semicircular) is not greater than  $7.878 \times 10^{-10}$ .

Perhaps some idea of these values may be obtained by noticing that the times of vibration of the small sphere under couple F per radian would be respectively 32 hours and 25 hours. But it is probably best to interpret the value in terms of the assumptions we made as to the force in the introduction. We found for the quadrantal couple

$$F = (G - G') MM'/r,$$
$$= \frac{G - G'}{G} \cdot \frac{GMM'}{r},$$

where MM' are the masses of the spheres, r the distance between their centres, GG' the parallel and crossed gravitation constants.

Now M, the mass of the larger sphere, is 399.9, say 400 grams,

M ,, ,, smaller ,, 1.004 grams,

r is 5.9 centims.,

G and G' are exceedingly near 6.66  $\times$  10<sup>-8</sup>, whence

$$\frac{\mathbf{G} - \mathbf{G}'}{\mathbf{G}} = \frac{\mathbf{F}r}{\mathbf{G} \cdot \mathbf{M}\mathbf{M}'} = \frac{1}{16500}.$$

On the assumed law of force this implies that the attractions between the two spheres, with distance 5.9 centims. between their centres, do not differ in the parallel and crossed positions by as much as  $\frac{1}{16500}$  of the whole attraction.

We may compare this result with RUDBERG'S values of the refractive indices of quartz for the mean D line

$$\frac{\mu_e - \mu_0}{\mu_0} = \frac{1.55328 - 1.544 \ 8}{1.54418} = \frac{1}{1.70} \text{ about.}$$

For the semicircular couple

$$2\mathbf{F} = \frac{\mathbf{G} - \mathbf{G}'}{\mathbf{G}} \cdot \frac{\mathbf{G}\mathbf{M}\mathbf{M}'}{r},$$

whence

$$\frac{\mathbf{G}-\mathbf{G}'}{\mathbf{G}} = \frac{\mathbf{1}}{\mathbf{2850}}.$$

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On the assumed law of force, this implies that the attractions between the two spheres, with distance 5.9 centims. between their centres, with their axes parallel and respectively in like and unlike directions, do not differ by as much as  $\frac{1}{2850}$  of the whole attraction.

This limit is large, undoubtedly owing to the want of axial symmetry in the apparatus which produced a semicircular couple as already pointed out. This couple was large, and though we attempted to eliminate it by the two sets of observations with the different azimuths of the larger sphere, in all probability we failed.

TABLE	I.—Showing Sc	eale-Readings in	Tenths of a	Division	aț I	Phases	at	$\operatorname{Heads}$	of
	Columns.	Time of Revo	lution of Lar	ger Spher	e 11	5 secs.			

0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
• •			• •	55	61	61	64	61	42
25	26	31	40	50	55	60	53	52	45
44	40	45	50	54	51	49	49	48	49
52	57	52	54	57	52	40	33	28	25
30	44	57	66	70	64	52	40	38	36
39	46	55	60	63	61	52	44	44	45
44	43	49	50	52	48	42	30	32	37
45	50	62	71	69	62	52	42	39	38
44	55	58	65	65	66	61	51	45	45
$\begin{array}{c} 41 \\ 48 \end{array}$	$\frac{38}{42}$	40	$\frac{49}{37}$	56	61	62	60 60	56	50
$\frac{48}{58}$	$\frac{42}{48}$	$39 \\ 41$	37 38	$\frac{40}{38}$	$\begin{array}{c} 42\\ 42\end{array}$	$\begin{array}{c} 58\\ 48\end{array}$	$\begin{array}{c} 69 \\ 54 \end{array}$	69 60	68 57
$50 \\ 55$	$\frac{40}{50}$	43	41	41	42	40	$\frac{34}{49}$	55	57 57
63	$60 \\ 60$	43 58	49	46	42	46	49 44	51	$57 \\ 52$
50	54	$\frac{38}{48}$	45	44	40	36	40	50	60
67	$67 \\ 67$	62	54	44	33	35	35	38	50
57	62	68	62	52	45	36	36	39	44
$\tilde{51}$	$\tilde{56}$	59	$\tilde{53}$	47	48	53	51	50	$\overline{49}$
50	49	52	50	50	51	53	52	54	55
48	47	44	41	44	52	55	58	60	56
49	41	41	42	43	47	50	55	60	60
60	56	58	47	43	47	49	50	50	50
54	54	54	48	50	51	49	52	52	45
42	43	48	49	55	56	52	52	52	57
56	51	46	42	42	43	49	51	55	55
55	52	49	57	50	50	50	44	43	50
50	50	43	43	46	50	58	54	55	50
49	49	49	48	50	51	54	53	56	56
$\begin{array}{c} 57 \\ 60 \end{array}$	58 60	56 60	$56 \\ 58$	$51 \\ 52$	$     43 \\     48 $	$\begin{array}{c} 40\\ 48\end{array}$	38     48	41	$\begin{array}{c} 51 \\ 57 \end{array}$
50	$\begin{array}{c} 60 \\ 60 \end{array}$	60 57	58 47	52 41	40	$51^{40}$	$62^{48}$	$\begin{array}{c} 52 \\ 63 \end{array}$	57
$\frac{50}{53}$	46	40	40	40	$41 \\ 43$	49		61	60
60	56	51	48	42	42	43	51	59	63
62	61	55	$52^{+0}$	51	50	51	51	$59\\52$	56
58	58	53	$45^{-0.2}$	40	41	49	60	70	70
60	52	48	48	50	50	$54^{10}$	55	53	51
50	50	47	50	50	52	53	53	50	48
48	46	48	50	51	51	50	50	52	52
49	46	44	44	49	50	55	59	57	58
51	49	46	43	44	51	59	68	64	56
50	42	40	49	57	68	71	70	59	50

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0.	1.	2.	3.	<b>4</b> .	5.	6.	7.	8.	9.
47	41	43	56	65	66	56	45	39	35
33	40	$\begin{array}{c} 43\\ 48\end{array}$	59	69	70	64	51	43	4.9
40	40	40	64	68 70	67	$\begin{bmatrix} 64\\56\end{bmatrix}$	40	39	90
$\begin{array}{c} 48 \\ 40 \end{array}$	51	$\begin{array}{c} 60\\52\end{array}$	04	70	07	50	42	59	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$
40	47	52	65	60	01	59		51	50
48	47	50	54	53	60	52	50	41	38
40	44	51	$egin{array}{c} 64 \\ 65 \\ 54 \\ 58 \\ 43 \end{array}$	$     \begin{array}{r}       60 \\       53 \\       66 \\       50     \end{array} $	67 61 60 70	71	63	50	38
35	$\begin{array}{c} 44\\ 38\end{array}$	$50 \\ 51 \\ 41$	43	50	56	59 52 71 70	$42 \\ 51 \\ 50 \\ 63 \\ 75$	70	59
<b>50</b>	46	45	51	61	70	71	70	62	52
$40 \\ 35 \\ 50 \\ 41 \\ 20$	40	$\frac{45}{40}$	$\begin{array}{c} 51 \\ 40 \end{array}$	47	$56 \\ 70 \\ 54 \\ 39$	$\begin{bmatrix} 60\\ 42 \end{bmatrix}$	70 71	71	68
60	50	48	45	39	39	42	49	51	60
	61	50		$ \begin{array}{c} 61 \\ 47 \\ 39 \\ 47 \\ 23 \\ 42 \\ \end{array} $	49	52	60	$62 \\ 71 \\ 51 \\ 72 \\ 57$	7
70	61	$50 \\ 57$	20	02	$     49 \\     21 $	30	4.9	57	77
04	02	63	52	40		94	-14	40	e c
01	79	03	01	42	00	01	40	49	
00	64	57	51	49	44		49	$49 \\ 52 \\ 42$	60 52 40
55	$\begin{array}{c} 55\\ 47\end{array}$	52	58	59	56	57	51	42	40
43	47	$52 \\ 55 \\ 49 \\ 48$	$51 \\ 51 \\ 58 \\ 61 \\ 59 \\ 53 \\ 42$	59 66 67 55	64	$ \begin{array}{c c} 34 \\ 47 \\ 57 \\ 60 \\ 56 \\ 57 \\ 57 \\ \end{array} $	52	50	45
41	45	49	59	67	67	56	50	49	43
38	45	48	53	55	56	57	55	54	56
53	49	42	42	51	61	69	70	65	54
45	41	$ \begin{array}{c} \overline{40} \\ \overline{60} \end{array} $	47	51	56	61	59	55	49
48	52	60	60	60	58	50	46	44	45 43 56 54 49 49
84665543413853454548454460	51	53 $49$ $63$	60	<b>51</b> 60 63 <b>52</b> <b>51</b> 48 <b>59</b> <b>5</b> 8 48 48 45	$\begin{array}{c} 44\\ 56\\ 64\\ 67\\ 56\\ 61\\ 56\\ 58\\ 67\\ 54\\ 41\\ \end{array}$	$ \begin{array}{c} 69\\ 61\\ 50\\ 62\\ 53\\ 39\\ 42\\ 52\\ 56\\ \end{array} $	$\begin{array}{c} 49\\ 60\\ 42\\ 38\\ 49\\ 51\\ 52\\ 50\\ 55\\ 70\\ 59\\ 46\\ 60\\ 50\\ 38\\ 47\\ 52\\ \end{array}$	55	4.8
44	46	49	$\begin{bmatrix} 50\\ 61 \end{bmatrix}$	52	54	53	50	$\begin{array}{c} 50\\ 50\\ 42\end{array}$	52
$\tilde{60}$	$\tilde{62}$	63	61	51	41	39	38	42	50
55	59	54	51	48	47	42	47	48	55
50	61	62	60	50	54	52	52	$\begin{array}{c} \overline{48} \\ 50 \end{array}$	50
00 50	54	55 - 55	55	50	56	56	50	51	51
55 58 56 52 56	54	00 81	55 52 50	10	$56 \\ 48 \\ 44$	54	50	$51 \\ 50 \\ 51$	51
50	$58 \\ 51$	$51 \\ 51$	52	40	40	04	00	50	51
52	51	51	50	45	44	42	40	51	55
56	53	56 46	$\begin{array}{c c} 59\\ 49\end{array}$	59	60	58	59	$59 \\ 57$	54
49	46	46	49	50	52	$54 \\ 42 \\ 58 \\ 58 \\ 58 \\ 58 \\ 58 \\ 58 \\ 58 \\ 5$	56	57	53
51	$50 \\ 62$	$\begin{array}{c} 50 \\ 51 \end{array}$	46	49	51	58	66	67	69
$51 \\ 65 \\ 62 \\ 55$	62	51	$\begin{array}{c} 46\\ 48\end{array}$	59     50     49     39     39	$\begin{array}{c} 60 \\ 52 \\ 51 \\ 39 \\ 38 \\ 53 \\ 72 \\ 73 \\ 51 \\ 58 \\ 49 \end{array}$	$     39 \\     47 $	50 55 46 59 56 66 45 62 42 52 61 52	67 51 67 38	56
62	$\begin{array}{c} 61\\52\end{array}$	$53 \\ 57 \\ 65 \\ 52 \\ 40$	48	4()	38	47	62	67	$\begin{array}{c} 4.8\\ 5.2\\ 5.0\\ 5.5\\ 5.0\\ 5.1\\ 5.1\\ 5.5\\ 5.4\\ 5.3\\ 6.9\\ 5.6\\ 6.3\\ 4.1\\ 4.0\end{array}$
55	52	57	56	56	53	49	42	38	41
51	60	65	56 71	56 73	72	60 73	52	50	40
42	49 38	52	$\begin{array}{c} 62\\ 43\end{array}$	71	73	73	$\overline{65}$	$\tilde{59}$	44
39	38	$\tilde{40}$	43	51	51	59	61	61	50
49	41	49	51	$51 \\ 52$	58	$59\\58$	52	52	50
51 42 39 49 53 49	56	57	$51\\51\\52$	$52 \\ 50$	40	$\frac{58}{49}$	$\frac{52}{49}$	51	$44 \\ 50 \\ 50 \\ 52$
40	$56 \\ 52$	53	51		••		~±J		
49				••	••	••	••	••	••
F.18F	r.1'00	F.149	F.100	r.040	r.004	r.955	F-004	5.000	50
5.175	5.163	5.143 min.	5.186	5.246	5.294	5.355	5.284	5.300	5.2
		mm.				max.			

TABLE I. (continued).—Showing Scale-Readings in Tenths of a Division at Phases at Heads of Columns. Time of Revolution of Larger Sphere 115 secs.

> Mean range 5.355 - 5.143 = 0.212 division. Greatest range in one period 7.5 - 3.5 = 4.0 divisions.

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Mean of 80 divisions.

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## DIRECTIVE ACTION OF ONE QUARTZ CRYSTAL ON ANOTHER.

## TABLE II.

Series.	tth of large ere.	l of revolu-	umber of periods observed.		Mean readings at phases (whole numbers omitted).								Mean range in Scale-	Greatest range in a period in	
	Azimuth sphere.	Period tion.	Number of observed	0.	1.	2.	3,	4.	5.	6.	7.	8.	9.	divisions.	Scale- divisions
A 1 A 2 B 1 B 2 B 3	° 0 180 180 180	secs. 115 115 115 115 115 115	80 80 80 70 54		$^{\cdot 163}_{\cdot 558}_{\cdot 590}_{\cdot 503}_{\cdot 632}$		$^{\cdot 186}_{\cdot 653}_{\cdot 648}_{\cdot 836}_{\cdot 648}$	246 813 556 941 656	·294 ·950 ·464 ·961 ·680	·355 1·030 ·379 ·843 ·717	$^{\cdot 284}_{1\cdot 008}$ $^{\cdot 328}_{\cdot 714}$ $^{\cdot 739}$	·300 ·929 ·284 ·540 ·785	·216 ·769 ·364 ·464 ·739		$ \begin{array}{r}     4.0 \\     3.1 \\     3.0 \\     7.5 \\     2.7 \end{array} $
C 1 C 2 C 3 D 1 D 2	0 0 0 90 90	secs. 230 230 230 230 230 230	72 80 80 50 70	$^{+708}$ $^{+370}$ $^{+616}$ $^{+024}$ $^{+031}$	$^{+731}_{-400}$ $^{+654}_{-654}$ $^{+042}_{-090}$	•708 •358 •686 1•034 •150	.739 .326 .673 1.004 .210	·717 ·271 ·663 ·988 ·220	·711 ·214 ·627 ·920 ·230	676 173 571 926 223	•678 •158 •560 •954 •176	$^{\cdot 642}_{\cdot 253}_{\cdot 566}_{\cdot 994}_{\cdot 126}$	·688 ·310 ·584 1·010 ·096		$     \begin{array}{r}       1 \cdot 3 \\       3 \cdot 4 \\       2 \cdot 0 \\       2 \cdot 2 \\       3 \cdot 1     \end{array} $

TABLE III.

a .	Mean readings at phases.										Mean
Series.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	range.
A	·414	·361	·355	·420	·530	$\cdot 622$	·693	·646	$\cdot 615$	·493	·338
B (advanced 180°)	·702	·646	·594	·536	·522	·519	·575	•631	·711	·718	·199
Means of A and B	·558	:503	·474	•478	·526	·570	·634	·638	·663	·605	·189
С	•565	·595	·584	·579	·550	·517	·473	·465	·487	·527	·130
D (advanced $90^{\circ}$ )	•575	•575	·565	•560	·553	·528	•566	·592	·607	·604	·079
Means of C and D	•570	·585	·575	·570	·552	·523	·520	·529	•547	•566	·065

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